Physics IA: Water cooling

Introduction

The goal of this investigation is to determine which means of cooling is the most efficient for cooling down a cup of boiling water. The idea for doing this investigation came about from the everyday situation of drinking tea and/or coffee that is too hot. Boiling water will cause damage to the mouth and throat if one attempts to drink it without cooling it down first. Studies have shown that 60 °C is the optimal temperature to consume hot beverages (Brown & Diller, 2008; Lee & O'Mahony, 2002). However, cooling the water down to this temperature can involve a lot of waiting, something which can be unwanted if one is in a hurry, or if the person drinking the coffee or tea wants to enjoy the beverage as soon as they can after making it. By comparing different means of cooling, this experiment identifies the most efficient cooling method to effectively solve the real-world issue presented above.

The experiment focuses on cooling methods which may be considered practical and also socially acceptable to perform in the context of cooling a cup of tea or coffee. As such, focus is primarily put on different ways of stirring, although some other cooling methods are used as well. Some cooling methods, such as lowering the cup into a box of liquid nitrogen, are considered very effective, but not practical to perform, so they have been omitted. Putting ice cubes into the water was also considered as a cooling method, but was omitted because it can be difficult to stop the cooling by the time the water reaches the desired temperature, and it is vital that the beverage is not *too cold* when consumed.

Research question

What is the fastest way to cool a cup of boiling water?

Variables

- The independent variable in this experiment is the method of cooling of the boiling water.
- The dependent variable is the rate of cooling of the water, identified by the overall change in temperature of the water during a predetermined measurement period.
- Controlled variables include the temperature of the environment, the temperature of the cup, the temperature of the water being poured into the cup, and the frequency of stirring (for cooling methods that require the water to be stirred repeatedly in a pattern of movements).

Chosen cooling methods

- **No interaction (method name: "Control")**

  Nothing was done to speed up the cooling of the water in the cup. The cup was placed in an idle atmosphere (i.e. no induced movements in the air) and was not interacted with. Heat dissipation was expected to occur as a result of evaporation and convection in the air above the cup, radiation from the cup, and conduction directly from the outer sides of the cup to the table and air surrounding the cup.

- **Putting a spoon in the cup (method name: "Spoon only")**

  Conditions were the same as in the control method detailed above, with the exception being that a spoon was placed in the cup prior to data collection. Heat dissipation was expected to increase slightly as a result of the spoon increasing the surface area of the heated body, and the spoon conducting the heat from the water into the colder air surrounding it.

- **Stirring in circles (method name: "Stirring CW")**

  The cup was placed in an idle atmosphere and a long spoon was inserted in the cup. The spoon was rotated clockwise continuously around the central axis of the cup. Heat dissipation as a result of evaporation and convection in air was expected to increase as a result of an increased water surface area from the rotational movement of the water itself, and from the formation of a vortex of air above the cup. An increase in convection in water was also expected due to increased movements within the water itself.

- **Repeated insertion and removal of the spoon (method name: "Repeated insertion")**

  The cup was placed in an idle atmosphere and a long spoon was repeatedly inserted completely and then removed completely from the cup. Heat dissipation due to radiation and conduction was expected to increase as a result of the warm spoon being exposed to the atmosphere directly for a fairly long time overall, aiding in quickly cooling the contents of the cup. A slight improvement in heat distribution within the water was also expected due to motion in the water incurred by the spoon.

- **Placing the cup in front of a fan (method name: "Fan")**

  The cup was placed in front of a fan producing a constant flow of air around the cup. No other interactions were done on the system. Heat dissipation as a result of convection in air was expected to dramatically increase.

- **Stirring back and forth (method name: "Back and forth")**

  The cup was placed in an idle atmosphere and a long spoon was inserted in the cup. The spoon was moved from one side of the cup to the opposite side, across the middle point of the cup, and back again. Heat dissipation was expected to increase as a result of better circulation of heat throughout the water (i.e. increased convection).

Materials and equipment

- A tea cup
- A spoon
- An electric table fan
- A measuring cup ($V = 500$ml, $\Delta V = \pm 10$ ml)
- A computer with Logger Pro
- A temperature probe
- A Vernier connection interface
- A boiling water source

Safety considerations

- Due to the dangers of working with hot water, care was taken to avoid getting skin in direct contact with the boiling water or the cup itself while it was hot. The handle of the cup was considered safe to touch.

Experimental procedure

The computer used for data collection was turned on and the temperature probe connected to the Vernier connection interface on port CH1. The interface was then connected to the computer using a USB cable, and Logger Pro was launched. The temperature displayed in Logger Pro was then read, indicating the ambient air temperature in the room where the experiment took place. The accepted temperature range was 24 ± 1°C. If temperature was outside this range, the room was heated or cooled as appropriate until the temperature was within the 24°C temperature range.

Each of the cooling methods were trialed 15 times in total during the experiment as described below. In order to avoid monotonous work, and to better avoid errors as a result of changes in the environment around the cup, two trials were done for each cooling method at a time. Two trials were performed for Stirring CW, then two for Control, followed by two for Repeated insertion, two for Spoon only, two for Back and forth, and finally two for Fan. This cycle was then repeated until 15 trials had been performed for each of the cooling methods.

For each trial, Logger Pro was configured to collect data for 630 seconds (11.5 minutes). The cup was filled with boiling water from the boiling water source. The water was left in the cup for a short time (approx. 10-15 seconds) before the cup was emptied, in order to pre-heat the cup. The cup was once again filled with boiling water, and the temperature probe immediately inserted into the cup. Once the temperature reading from Logger Pro stopped increasing, and if the temperature at that point was higher than 95°C, the chosen cooling method's procedure (outlined under "Chosen cooling methods" above) was initiated. If the temperature reading was below 95°C, the cup was emptied and filled again with boiling water from the boiling water source, repeated until the temperature of the water eventually exceeded 95°C. As soon as the temperature reading dropped below 95°C, data collection was started in Logger Pro. Cooling was done according to the procedure until the 630-second data collection window had passed, and data collection was automatically stopped. The water from the cup was then disposed of in a sink.

Preliminary adaptations

Preliminary adaptations of the data was done in order to standardize and reduce the amount of data that would be analyzed in this report. The performed changes were as follows:

- $t = 0$ was defined as the first point at which each trial dropped to immediately below 94°C. All data prior to this point was omitted for each trial. This was done to provide a common starting point for all of the data.
- The trial with the fewest remaining data points after the first instance of $T < 94°C$ was identified. This was found to be Control, trial 2, with the final data point having been recorded at $t = 588.5\text{s}$ (1178 data points in total). All other trials in the experiment were then truncated at $t = 588.5\text{s}$ so that all of the trials had an equal amount of data points.

Raw data tables

Raw data from individual trials have been omitted. Instead, the table below displays the <u>averaged</u> values from all of the trials for each cooling method.

Time [s]	Temperature at given time (for each cooling method) [± 0.1 °C]					
	Stir. CW	**Control**	**Rptd.ins.**	**Spoon**	**Back&fth.**	**Fan**
0	94.0	94.0	94.0	94.0	94.0	94.0
20	92.0	92.6	92.3	92.7	92.4	90.7
40	90.4	91.4	90.7	91.5	91.0	87.8
60	88.9	90.2	89.2	90.4	89.7	85.3
80	87.5	88.9	87.9	89.3	88.5	82.9
100	86.2	87.9	86.6	88.1	87.4	80.9

120	84.9	86.8	85.5	87.2	86.2	78.8
140	83.7	85.8	84.3	86.2	85.2	77.0
160	82.6	84.8	83.1	85.2	84.2	75.2
180	81.5	83.9	82.0	84.4	83.2	73.6
200	80.5	83.1	81.0	83.6	82.2	72.1
220	79.6	82.3	80.0	82.6	81.3	70.7
240	78.6	81.4	79.1	81.9	80.5	69.2
260	77.8	80.6	78.1	81.1	79.6	68.0
280	76.9	79.9	77.3	80.3	78.8	66.7
300	76.1	79.2	76.4	79.6	78.0	65.6
320	75.3	78.4	75.6	78.9	77.3	64.4
340	74.5	77.8	74.8	78.2	76.6	63.3
360	73.8	77.0	73.9	77.5	75.8	62.3
380	73.0	76.4	73.2	76.9	75.1	61.3
400	72.3	75.7	72.4	76.3	74.4	60.3
420	71.7	75.1	71.8	75.6	73.8	59.4
440	71.0	74.5	71.0	75.0	73.1	58.5
460	70.4	73.9	70.4	74.4	72.5	57.6
480	69.7	73.4	69.7	73.9	71.9	56.8
500	69.1	72.8	69.1	73.3	71.3	56.0
520	68.5	72.2	68.4	72.7	70.7	55.2
540	67.9	71.7	67.8	72.2	70.1	54.5
560	67.4	71.2	67.3	71.7	69.6	53.8
580	66.8	70.6	66.7	71.2	69.0	53.1

Data visualizations

Figure 1 shows a graphical representation of the average temperature data collected over the 588-second measurement period for each cooling method.

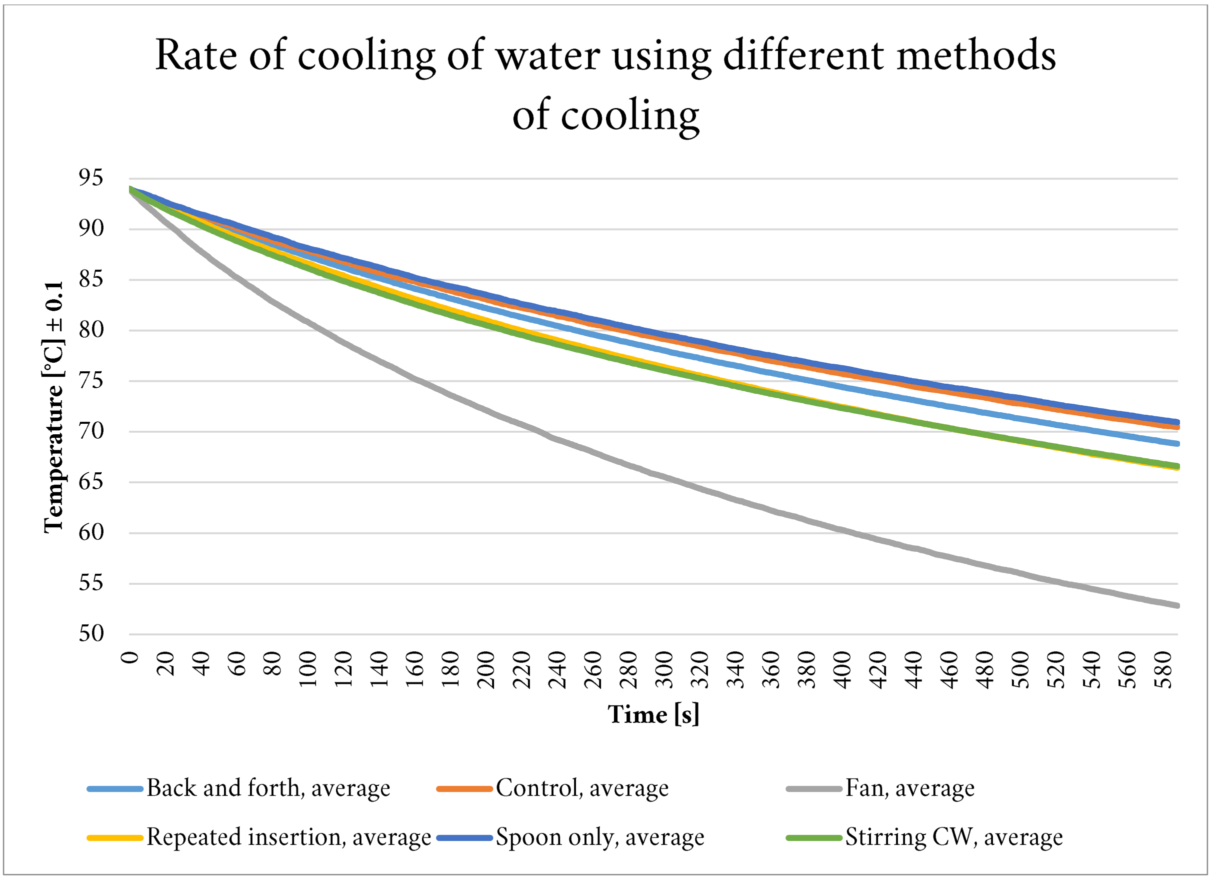

Figure 1: Change in water temperature over time

Data analysis

In order to determine which of the given cooling methods is the fastest for cooling down water, it is necessary to find a data model that fits the collected data. Given that this is an experiment on cooling, it is natural to look at Newton's Law of Cooling, and to try and find an expression that can fit the collected data. Newton's Law of Cooling states that the rate of change of temperature of a body is proportional to the difference in temperature between the body and its surroundings (Emmons, 2016). A general function exists to describe the temperature of the body after a certain amount time has passed:

$$T(t) = T_S + (T_0 - T_S)e^{-kt}$$

Where:

- $T(t)$ is the temperature of the cooling body at any given time t
- T_S is the temperature of the surroundings
- T_0 is the initial temperature of the body
- t is the time that has passed since the body was at temperature T_S
- k is the cooling constant

The cooling constant is a number that describes how fast the body cools. For example, a water droplet will have a much higher cooling constant than a bucket full of water, as the droplet cools down much faster. Knowing the cooling constant of each of the cooling methods used in this experiment allows one to obtain specific numbers that indicate how fast each of the methods can cool water. The function can be rearranged to find this constant:

$$k = \frac{\ln \frac{T(t) - T_S}{T_0 - T_S}}{-t}$$

This equation was applied to the raw data to obtain the cooling coefficient for each averaged sample from each cooling method, with the following values:

- $T_S = 24.0\ °C$
- $T_0 = 94.0\ °C$

An example of such a calculation is included here for the Control method at $t = 300$:

$$k = \frac{\ln \frac{78.0 - 24.0}{94.0 - 24.0}}{-300} = \frac{\ln \frac{54.0}{70.0}}{-300} = \frac{\ln 0.771}{-300} = \frac{-0.260}{-300} = 8.65 \times 10^{-4}$$

The resulting data was then graphed as a function of time t:

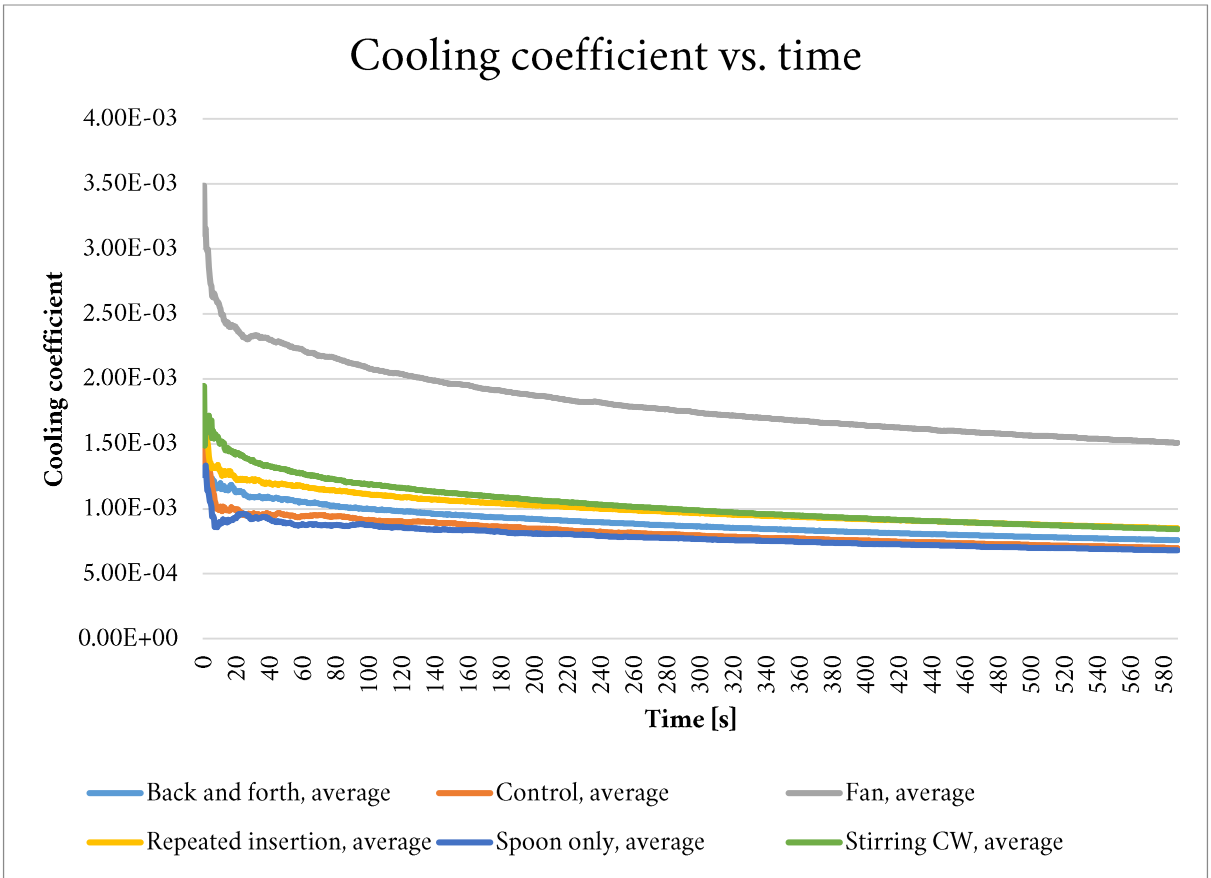

Figure 2: Change in cooling coefficient over time

Interestingly, there is a significant peak in these values at the beginning of the data. Figure 3 shows a zoomed-in version of Figure 2 when $t < 60$:

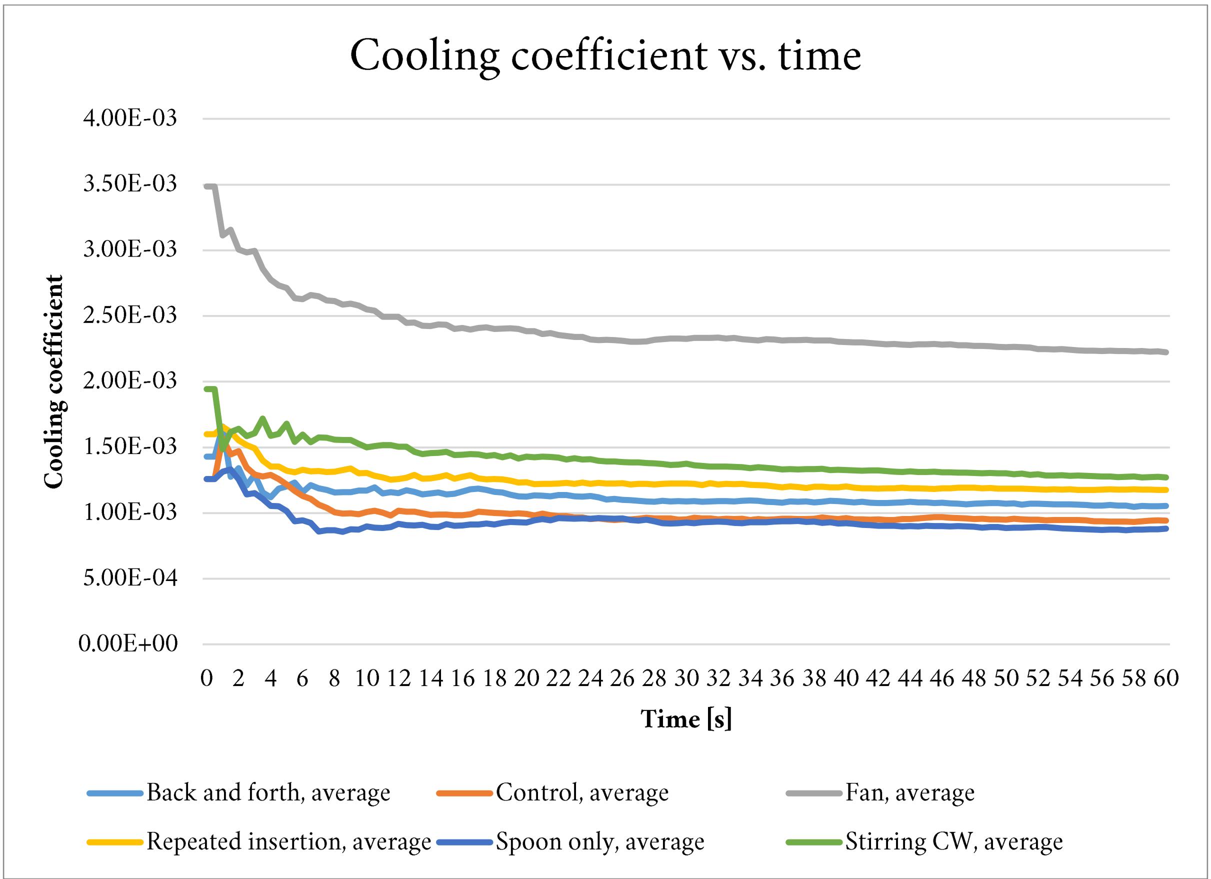

Figure 3: Change in cooling coefficient over time, first 40 seconds of experiment

The data shows that k, which was supposed to be constant in Newton's Law of Cooling, was fluctuating, starting at very high values and gradually declining over time. This finding was very surprising, indicating a potential undiscovered error source. Suspecting that the data from the start of the experiment was affected by experimental errors, such as the tea cup or the spoon potentially not having reached thermal equilibrium with the water at such an early stage of the trial, the first 100 seconds of the collected data was discarded, but the cooling coefficients were still declining over time even after that data had been omitted. The causes of errors here are further discussed in the section on error sources.

Other potential data models were then explored to identify something that would accurately represent the collected data. As it turned out, a hyperbolic model produces graphs that are almost perfectly linear when applied to data for the temperature difference between the cup and the ambient temperature of the room. A multiplicative inverse transformation was applied to the collected data after the ambient temperature 24.0 °C was subtracted from all values, and the results were graphed as a function of reciprocal temperature over time. The resulting graphs were easy to fit to a linear growth model, and the slope of these graphs could be used to determine which cooling method was the most efficient.

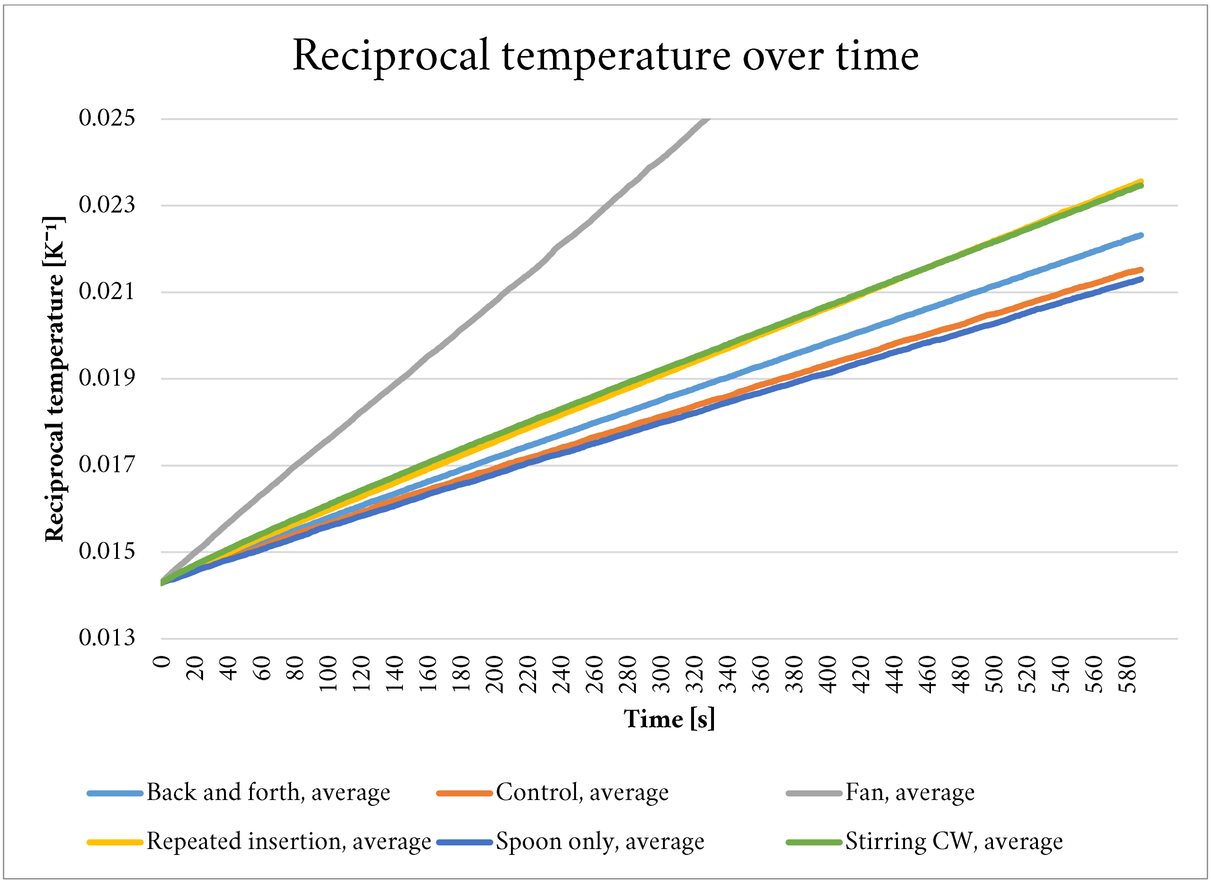

Figure 4: Hyperbolic growth model

While a hyperbolic growth model can only be considered an approximation for the actual function of cooling over time, the model is accurate enough to distinguish between the cooling efficiency of the various cooling methods by looking at each graph's slope, and the obtained values indicate directly how efficient each of the cooling methods were in relation to each other.

Cooling method	**Slope [$K^{-1}\ s^{-1}$]**
Back and forth	1.35×10^{-5}
Control	1.22×10^{-5}
Fan	3.42×10^{-5}
Repeated insertion	1.56×10^{-5}
Spoon only	1.18×10^{-5}
Stirring CW	1.53×10^{-5}

When these values are plotted on a bar chart, it becomes immediately clear which cooling method is the fastest. By the very nature of reciprocals, a higher reciprocal value reflects a lower actual value. Hence, by looking at which method produces the steepest upwards reciprocal slope, the cooling method that produces the fastest rate of change in temperature can be identified – see figure 5.

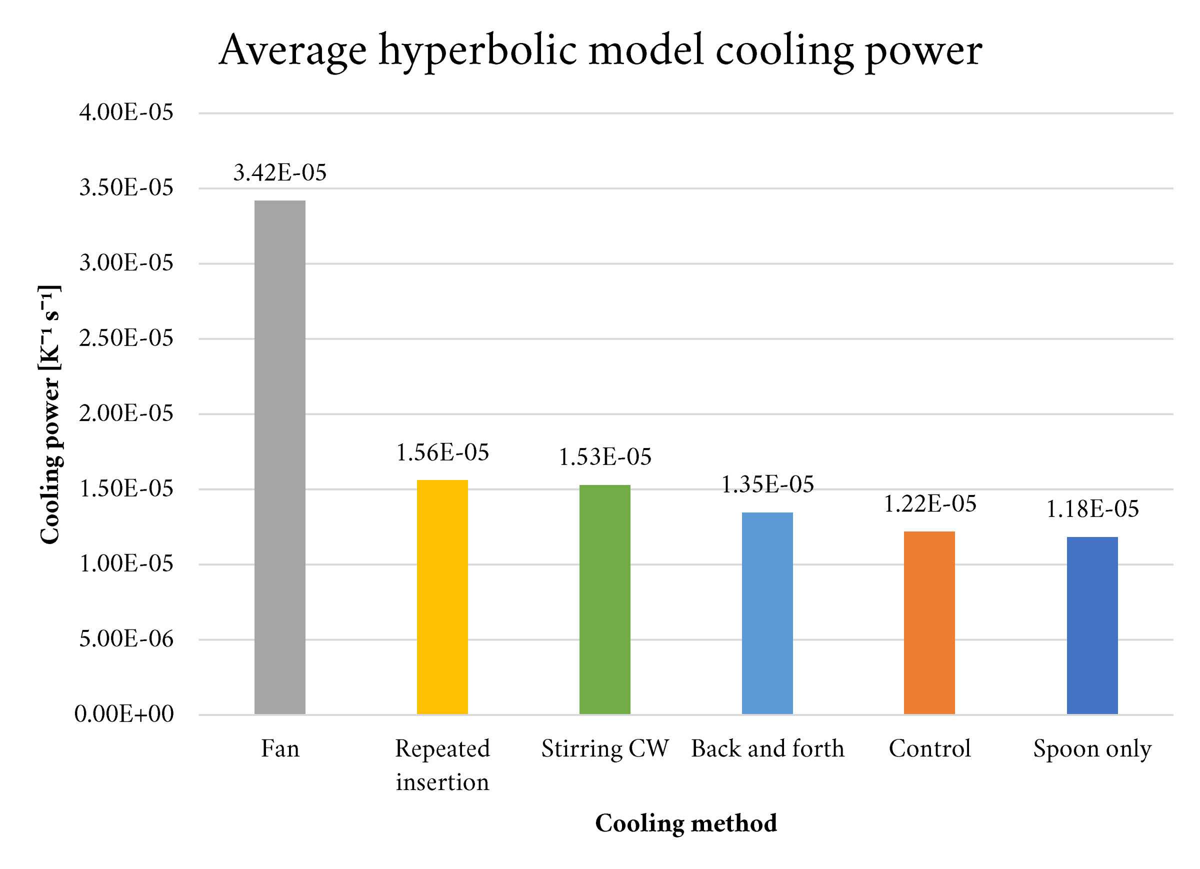

Figure 5: Cooling power according to the hyperbolic model, sorted highest to lowest

Conclusion

The results of the experiment indicate that placing the cup of water in front of a fan is the fastest way to cool it down. This method is approximately twice as fast as any of the other methods and was the only method that managed to reach the recommended drinking temperature of hot beverages, 60 °C, during any of the trials, reaching it on average after 407 seconds.

However, a fan might not always be available for cooling down tea, and since fans have a tendency to be noisy, and since it would by many be considered odd to use a fan to cool down a hot beverage in a social setting, it is also worth looking at the results that do not involve usage of a fan. The differences between the other methods of cooling can be considered small, with only 4.5 °C separating the fastest cooling method from the slowest at the end of the collected data. Repeatedly inserting and removing a spoon in the cup seems to be very marginally faster than stirring in circles in the cup, but this difference may for all practical purposes be considered negligible, and may even be caused by experimental errors, given that the difference between these two cooling methods is so small.

As part of the Control and Spoon only methods, no movement was induced in the water or in the air around the cup to actively cool it, and these two methods were, as expected, the slowest methods for cooling the water. Placing a spoon in the cup seems to have marginally *slowed down* the rate of cooling when compared to the control trials. It is speculated that this may be because of the spoon adding mass to the cup of water, but it could also have been due to random errors, i.e. inconsistencies in the temperature of the environment between the trials. Moving the spoon back and forth across the cup yielded a cooling effect somewhere in between the stirring method and the control method, which was also as expected.

Error sources

Several error sources were observed during the experiment which may have had an impact on the collected data.

- The ambient temperature in the room may have fluctuated throughout the experiment. This is a systematic error for all of the data points within the affected trials, but between trials, it is considered a random error, because all of the methods were performed under each condition due to the cycling between methods described in the experimental procedure, changing the error from being between methods to being between trials. This error would result in the liquid cooling slower or faster, depending on if the ambient temperature at the time of data collection is higher or lower than the average.
- The tea cup was an open system. This led to the evaporation of water out of the system, reducing the mass of the system and potentially causing a temporarily increased cooling power in the early stages of the experiment. This could explain why the cooling constant in Newton's Law of Cooling was fluctuating over time. Evaporation would also diminish as the temperature of the cup dropped, explaining the gradual decline of the cooling coefficient in the mid and late stages of the experiment.
- The thermal energy of the tea cup was dissipated into the environments in several ways in which Newton's Law of Cooling applies poorly or not at all. Newton's Law of Cooling is accurate for thermal conduction, but can only approximate convective heat transfer, and does not apply to thermal radiation. Convective heat transfer was one of the largest causes of heat dissipation in the experiment, and thermal radiation also had some effect. This could also explain the

fluctuations in the cooling coefficient values in conjunction with the previously stated error of the cup being an open system.

Suggested improvements

Several steps could be taken to reduce the impact of the errors outlined above during the experiment. The first step would be to perform the experiment in an environment in which ambient temperature can be fully controlled. This would reduce the systematic and random errors associated with ambient temperature fluctuations.

The cup could be covered at the top to turn it into a closed system to reduce the impact of mass loss from evaporation on the fluctuations of the cooling coefficient in Newton's Law of Cooling. This was intentionally not done in this experiment, because several of the chosen cooling methods rely on an increase in convective heat transfer in the air above the cup to cool the cup faster. Aside from beverages bought in coffee shops, restaurants and similar establishments, it is also unusual for a cup to be covered at the top when drinking hot beverages. A cover would also inhibit the potential for stirring.

Works cited

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